In this activity you will use natural logarithms to model data about a smoke layer above a fire in a building.

Information sheet

In tall building spaces, smoke from a fire at the bottom can form into a stagnant layer before it reaches the ceiling or roof of the space. Formation of such layers is called stratification.

Stratification occurs when there is an increase in ambient temperature from the bottom to the top of the building. This is often the case in tall buildings with an atrium. The difference could be due to a combination of conditions within the building, air conditioning effects, or sunlight shining through a glass roof. The danger is that stratification effectively prevents smoke from rising high enough or quickly enough to reach ceiling mounted smoke-detectors until the fire has reached conflagration stage.

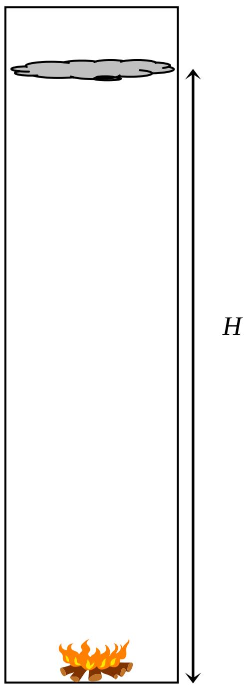
The height of the smoke layer above the fire depends on the temperature difference between the bottom and top of the space, and also the heat output of the fire.

The following table gives values for smoke tests carried out with a controlled 5 kW and a 10 kW fire.

H metres is the height of the smoke layer above the bottom of the building space measured correct to the nearest 0.5 m.

T °C is the difference in ambient temperature between the bottom and top of the building space.

	H	
T	5 kW fire	10 kW fire
2	19.5	25.5
4	13	17
6	10	13.5
8	8.5	11
10	7.5	10
12	6.5	9
14	6	8
16	5.5	7.5



Think about…

As the temperature difference increases, does the smoke layer rise or fall?

Do you think a graph of H against T would give a straight line?

What techniques could you use to change a curved graph into a straight line?

Try these

Throughout this question refer to the **Information sheet**.

a For the 5 kW fire the height, H metres, at which the smoke layer will form when the temperature difference between the top and bottom of the building is T °C. This can be modelled by the equation $H = aT^n$, where a and n are constants.

Show that $\ln H = n \ln T + \ln a$

b i Complete the rows in the table, giving values of $\ln T$ and $\ln H$ for the 5 kW fire.

T	2	4	6	8	10	12	14	16
H	19.5	13	10	8.5	7.5	6.5	6	5.5
$\ln T$								
$\ln H$								

ii Draw a graph of $\ln H$ against $\ln T$.

iii Explain how the graph shows that $H = aT^n$ is an appropriate model.

iv Use your graph to show that $H = 30T^{-0.6}$ would be a suitable model for the 5 kW fire.

c Use the model $30T^{-0.6}$ for the 5 kW fire to:

c i predict the height of the smoke layer which would arise when the temperature difference between the bottom and top of the building was 3°C

c ii ***sketch*** a graph of H against T

c iii explain why the prediction made by the model for the case when the temperature is the same at the bottom and top of the building is inappropriate.

d For the 10 kW fire, the relationship between H and T can be assumed to be $H = bT^{-0.6}$ where b is a constant.

i By substituting the values $H = 25.5$ and $T = 2$ into this equation, find an estimate of the value of b.

ii Explain briefly why the value for b that you have found in part **d i** is unlikely to be as accurate as the value of a found in part **b iv**.

Reflect on your work

Describe how you would use a spreadsheet to draw an appropriate graph to find the values of the constants for the 10 kW fire.

Explain why it is appropriate in this context to draw a log–log graph, but not a log–linear graph.